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DETERMINATION OF MOLECULAR SYMMETRIES BY FORCE FIELD CALCULATIONS AND EVALUATION OF SYMMETRIC AND NONSYMMETRIC CONFORMATIONAL TRANSITION STATES AVOIDING COMPLETE POINT-BY-POINT MAPPING

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Abstract—In connection with the application of force field calculations to the derivation of molecular symmetries some remarks of caution are given. Frequently used energy minimisation procedures cannot decrease molecular symmetries; only an increase of symmetry can occur. This phenomenon can be used for the calculation of symmetric conformational transition states without using point-by-point mapping techniques. A procedure for the economic calculation of general conformational transition states is described which avoids exhaustive mapping of interconversion paths.

In the course of the minimisation of molecular energies as given by a force field the question often arises as to the molecular symmetry. We wish to point out that by application of widely used minimisation techniques no molecular symmetry elements can disappear in the course of the calculation. Caution is therefore demanded when deriving molecular symmetries in this way. Additional symmetry elements can, however, be generated. This phenomenon may be used for the calculation of properties of certain symmetric conformational transition states. In a further section we show how general symmetric and nonsymmetric conformational transition states can be accurately calculated without using exhaustive mapping techniques, such as described by Wiberg and Boyd.[1]

(a) *Conservation of symmetry elements during energy minimisation.* We start our discussion by considering as a simple example the water molecule of symmetry C_{2v} with the following Cartesian atomic coordinates for the two H atoms and the O atom (x_i; for clarity we denote the individual components by x, y and z): 0, y_1, 0; 0, $y_3(=-y_1)$, 0; 0, 0, z_2. If we apply a steepest descent procedure such as given in Eqn (1), in which all the

$$\delta x_i = -k \frac{\partial V}{\partial x_i}; \quad \begin{matrix} V \text{ is the potential energy,} \\ k \text{ is a scaling constant} \end{matrix} \qquad (1)$$

coordinate shifts δx_i are calculated simultaneously from the derivatives $\partial V/\partial x_i$ (all evaluated for the same molecular geometry of a particular cycle), the following relationships must hold throughout the minimisation: $\delta y_3 = -\delta y_1$ and $\delta z_3 = \delta z_1$. Furthermore, all the other coordinate shifts, except δz_2, must be zero throughout the steepest descent iterations because of zero respective derivatives of V. (Due to symmetry the energy function is even with respect to these coordinates around the corresponding atomic positions.) It follows that the C_{2v}-symmetry elements are conserved during the minimisation. With some fictituous force field (e.g. containing a HOH-reference angle of 180°) new symmetry elements could, however, be generated such that a molecular point group would result of which C_{2v} is a subgroup ($D_{\infty h}$ for our example) since the above relationships hold also for a supergroup of C_{2v}.

Similar arguments for systems of arbitrary complexity and for arbitrary choice of the coordinate system and the coordinate type (Cartesians or internals) lead to the following general conclusions: The molecular symmetry is reflected in the first and second (and higher) partial derivatives of V. Hence in all minimisation techniques with simultaneous calculation of the δx_i (all derivatives calculated for the same molecular geometry) no symmetry elements can disappear. In particular this holds also for the Newton–Raphson procedure (Eqn (2)) which is now

$$\delta \mathbf{x} = -\mathbf{F}^{-1} \mathbf{grad}\, V; \quad F_{ij} = \frac{\partial^2 V}{\partial x_i \partial x_j} \qquad (2)$$

frequently used as a highly efficient tool for accurately locating energy minima. Additional symmetry elements can, however, be generated by these methods.

The general necessary condition for an energy minimum is the validity of the set of equations

$$\partial V/\partial x_i = 0. \qquad (3)$$

This condition is of course not sufficient since it holds also for partial maxima and inflection points with zero slope. (We use the term "partial maximum" rather than "saddle-point"; a saddle-point (transition state) is a onedimensional partial maximum. For a more detailed discussion of this subject the reader is referred to a paper of McIver and Komornicki[2] which deals with quantum-mechanical calculations of transition states. A onedimensional partial inflection point with zero slope corresponds to a pseudorotational degree of freedom.) The commonly used energy minimisation procedures only lead to a satisfaction of Eqn (3) but do not automatically take into account necessary *and* sufficient conditions for minima. Because of conservation of symmetry elements we will calculate a symmetry-constrained minimal energy structure satisfying Eqn (3) when applying the minimisation procedures mentioned to an exactly symmetric trial structure. This structure need not correspond to an energy minimum, but can also correspond to a partial maximum or a partial inflection point with zero slope. The presence of a M-dimensional partial maximum leads to M negative

eigenvalues (i.e. to 3N-6-M positive nonzero eigenvalues for a N-atomic molecule) of **F** (and also to negative diagonal elements F_{ii}), and can therefore easily be detected if one uses the Newton–Raphson procedure. (A Z-dimensional partial inflection point with zero slope leads to Z extra zero eigenvalues in addition to the six vanishing eigenvalues for translation and rotation. The **F**-matrix for a molecule at a true global minimum must of course have 3N-6 positive eigenvalues and six zero.) For the cases of interest M is a small number, often one. Some cases with $M > 1$ we have encountered are mentioned below.

If one wants to determine molecular symmetries by force field calculations one should always as a measure of caution start the energy minimisations with trial structures of sufficiently low symmetry, and the less symmetric distortion should not be too small (*cf.* Section *c*). Two examples are given for illustration. The two possible reasonable symmetries for *cis*-butene-2 are C_2 (twisted Me groups) and C_{2v} (eclipsed Me groups). Using our olefin force field[3] gives C_{2v} as a true minimum. The energy minimisation (steepest descents followed by Newton–Raphson iterations) was started with a C_2-symmetric trial structure containing highly twisted methyl groups. For *cis,cis* - cyclooctadiene - 1,5 also C_2- and C_{2v}-symmetries require consideration. Energy minimisation with the same force field of an exactly C_{2v}-symmetric structure leads to one negative eigenvalue of **F** (corresponding to an imaginary frequency of $166{\cdot}2\sqrt{(-1)}\,cm^{-1}$). The adjacent true minimum ("twisted boat"[4]) has only C_2-symmetry with calculated $C\text{–}CH_2\text{–}CH_2\text{–}C$ torsion angles of 52·5°. (These torsion angles are zero for the C_{2v}-structure; see also Section b.) Special caution is generally called for if the structure in question contains zero torsion angles around CC-single bonds required by intersecting perpendicular mirror planes. We have found a number of cases in the literature which in the light of these explanations make us feel that the respective energy "minimisations" have been started (and therefore also finished) with symmetric structures containing partial maxima.*

The difficulties described for the minimisation of the energy of symmetric trial structures may, in principle, be overcome without initial distortion of the molecular geometry by applying steepest descent techniques in which the derivatives are recalculated after shifting each atom separately.[9,10] The author has no practical experience as to the performance of these techniques.

(b) *Calculation of symmetric transition states surrounded by minima of lower or different symmetries.* The troubles encountered when minimising the energy of non-minimum symmetric molecular trial structures can sometimes be turned into an advantage if the symmetric structure is a model for a conformational transition state, and one is interested in the calculation of its properties. To this end the transition state model must contain additional symmetry elements as compared to the adjacent minima, or else must possesss symmetry elements different from those of the adjacent minima. It is sufficient for the discussion to give a few examples. Wiberg and Boyd have applied their point-by-point mapping technique[1] to the calculation of the transition state for the inversion of the sixmembered rings in bicyclo(3.1.1)heptane. A good model for this transition state is a C_{2v}-symmetric structure which contains an additional mirror plane (and a twofold axis) as compared to the two adjacent minima of C_s-symmetry. Steepest descent and Newton–Raphson iterations applied to an exactly C_{2v}-symmetric trial structure of this compound will therefore yield the transition state without mapping. The above mentioned C_{2v}-symmetric boat structure of *cis,cis* - cyclooctadiene - 1,5 is a model for the transition state connecting two enantomeric C_2-minima. We calculate its optimum potential energy 7·09 kcal mole^{-1} above the C_2-minima (5·39 including vibrational contributions†)‡. The eigenvector corresponding to the imaginary frequency of this C_{2v}-structure represents the reaction coordinate for the interconversion process at the transition state (see below). As expected, it shows the symmetry of an A_2-mode and has large components for the $C\text{–}CH_2\text{–}CH_2\text{–}C$ torsions. Similar examples for the calculation of symmetric partial maxima are the cyclohexane boat form (C_{2v}) as compared to the twist-boat forms (C_2, D_2) or the chair (D_{3d}), eclipsed ethane (D_{3h}) as compared to the staggered minimum (D_{3d}), or planar cyclobutane (D_{4h}) as compared to the puckered structure (D_{2d}).

(c) *Calculation of general conformational transition states without exhaustive mapping.* The basis of the points raised in the following is partly elementary text-book knowledge. The purpose of this section is to hint at consequences relevant for the calculation of conformational interconversion barriers.

We shall outline the procedure by describing its application to the calculation of a barrier corresponding to a nonsymmetric transition state between two *trans*-cyclooctene conformations which are shown in Fig. 1 together with calculated internal parameters. Conformation A of Fig. 1 is a distorted chair conformation (symmetry C_2), B a nonsymmetric less stable conformation. The interconversion between both conformations is a part of the more complex interconversion path between the crown conformation (stable *trans*-cyclooctene conformation[11]) and conformation A. This latter process proceeds via several barriers the calculated highest of which (T, Fig. 1) turns out to be that connecting the minima of A and B. The calculation of the minimum energy path of the crown-distorted chair interconversion is of interest in connection with experimental conclusions as to the stable *trans*-cyclooctene conformation;[11] a complete description of the calculations will be given elsewhere.

The calculated potential energy profile for the interconversion of A to B, partly obtained by a mapping technique similar to that of Wiberg and Boyd[1] is given in Fig. 2 along

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*See Appendix A.

†Potential energy differences ΔV incremented by vibrational contributions ΔH_{vibr} represent enthalpy differences ΔH: $\Delta H = \Delta V + \Delta H_{vibr}$. H_{vibr} is given by $RT \sum_{i=1}^{S} X_i(\frac{1}{2} + 1/[\exp X_i - 1])$ with $X_i = h\nu_i/kT$. The summation extends over the S real nonzero frequencies. ($S = 3N-6$ or $3N-7$ depending on whether the conformation corresponds to a minimum or a onedimensional partial maximum.) In the case referred to here $\Delta V + \Delta H_{vibr}$ is the activation enthalpy ΔH^+ of the conformational interconversion process. The H_{vibr}-values were calculated for $T = 298{\cdot}2°K$ throughout.

‡The C_{2h}-symmetric chair conformation is calculated as a true minimum with a potential energy 4·09 kcal mole^{-1} above the C_2-conformation ($\Delta H = 3{\cdot}09$ kcal mole^{-1}). Allinger and Sprague,[5] described the C_{2v}-symmetric boat as a minimum and calculated it 1·79 kcal mole^{-1} (no vibrational contributions) less stable than the C_{2h}-symmetric chair with the help of a modified steepest descent technique. The C_2-form was not considered by these authors.

Note added in proof: Allinger and Sprague have recently reconsidered the problem (*Tetrahedron* 31, 21 (1975)) and now describe the C_{2v}-form as a transition state between two C_2-forms.

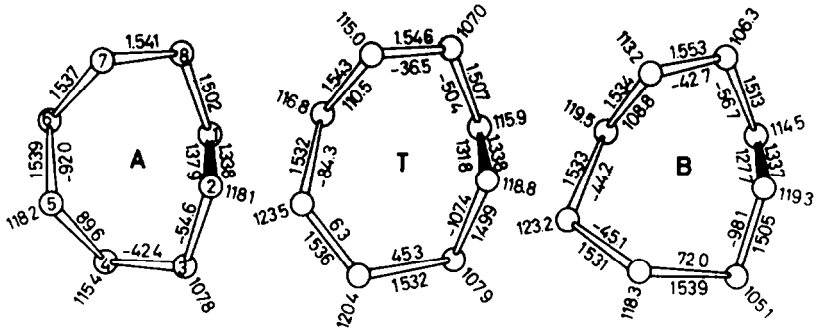

Fig. 1. Calculated geometries of conformations A (symmetry C_2) and B (no symmetry) of *trans*-cyclooctene, and of the transition state T (no symmetry) relating these two conformations (peripheral values: bond lengths and angles (Å, deg); others: torsion angles (deg)).

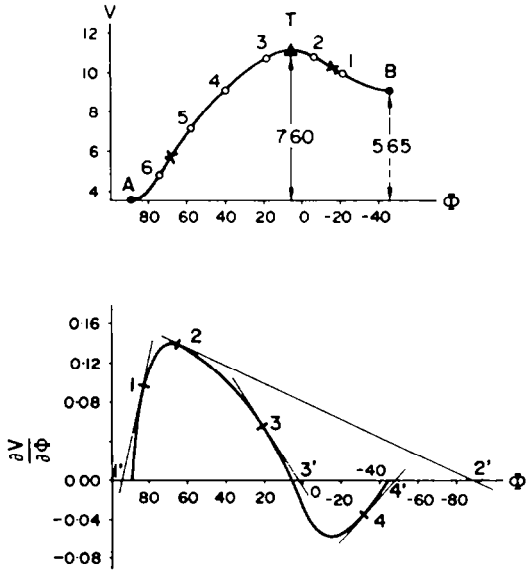

Fig. 2. Top: Calculated potential energy profile for the interconversion of conformations A and B of *trans*-cyclooctene (Fig. 1). V: potential energy (kcal mole^{-1}) relative to crown conformation (conformation A 3·54 kcal mole^{-1} (no vibrational contributions) less stable than crown conformation). Φ, torsion angle C(3)–C(4)–C(5)–C(6) (deg); ○, ●, ▲, ×, mapping points (numbering according to the sequence of their calculation), minima, partial maximum, and inflection points, respectively. Bottom: first derivative (kcal mole^{-1} deg^{-1}) of upper profile with respect to the torsion angle C(3)–C(4)–C(5)–C(6) (see text for further details).

with the curve of its first derivative.* The torsion angle C(3)–C(4)–C(5)–C(6) (Φ; Fig. 1) was assumed as the reaction coordinate. (Strictly speaking, this parameter is not the complete reaction coordinate but only an important component of it.) This torsion angle is not symmetrically disposed with respect to the twofold axis of A, and the addition of a constraint-potential for this torsion angle will therefore destroy the corresponding symmetry of the total potential describing A. Thus the mapping can proceed from A without encountering difficulties such as mentioned in Section a. (If for a mapping calculation of the kind described here a torsion angle symmetrically disposed with respect to a twofold axis of symmetry is assumed as the reaction coordinate and the transition state in question does not possess this twofold symmetry the starting geometry has to be distorted according to the criteria discussed in Section a and further below.)†

In the curve for the derivative of the profile (Fig. 2) four tangents are drawn for a graphical demonstration of how the Newton–Raphson procedure operates. The transition state corresponds to a onedimensional partial maximum, and a discussion in this single dimension suffices. The energy is at minimum for all the other 3N-7 degrees of freedom (N = 22). We discuss what happens if we now start a series of *normal unconstrained* Newton–Raphson iterations (Eqn 2) *without* preceding steepest descent cycles (Eqn 1) from the four points 1, 2, 3, 4; Fig. 2, bottom) where the tangents touch the curve of the derivative. Starting from 1 and 4 will lead to 1′ and 4′, and eventually to the left and right minimum, respectively. Starting from 2 will cause a very drastic change of the C(3)–C(4)–C(5)–C(6) torsion angle to 2′, and there will be very probably no convergence since some of the partial derivatives with respect to degrees of freedom other than the reaction coordinate will be two far from zero at the new value 2′ of the reaction coordinate. Starting the Newton–Raphson iterations from 3, on the other hand, will eventually lead with high precision to the partial maximum of the transition state.

Summarising in more general terms we conclude: Newton–Raphson iterations (Eqn 2) lead to a satisfaction of Eqn (3) irrespective whether a minimum or a maximum is approached, while properly scaled steepest descent cycles always (excluding, of course, structures which precisely correspond to a partial maximum; see below for convergence properties) lead to a lowering of V. (This latter property refers only to steepest descent procedures which calculate the coordinate shifts according to Eqn (1). Procedures which use diagonal or block diagonal[10] approximations of **F** for scaling are essentially of the Newton–Raphson type and have the same properties important in our context as those of full-matrix Newton–Raphson techniques. For the algorithm using **F** approximated by its diagonal (k in Eqn (1) replaced by a vector **k** with $k_i = F_{ii}^{-1}$) it is particularly easy to see how maxima are approached since in this process F_{ii}'s are negative as mentioned in Section a.) Thus for calculating general symmetric and nonsymmetric maxima of conformational transitions the starting geometry has to be within the geometries of the maximum and the adjacent inflection points without being too similar to the geometry

*See Appendix C for details of the minimisations.
†See Appendix B.

of the latter, and only Newton–Raphson iterations are to be applied without preceding steepest descent cycles. (Conversely one can involuntarily end up on a maximum if one starts too close to the maximum and does not apply enough steepest descent cycles before reverting to Newton–Raphson iterations. Furthermore it is to be taken into account that the convergence properties of the steepest descent process are similar for minima and maxima in the sense that abandoning a maximum is as slow as approaching a minimum.) The procedure is to some extent a generalisation of the method outlined in Section b for the calculation of maxima surrounded by minima of lower or different symmetries. (It is therefore clear that also if one wants to get rid of a symmetric maximum the less symmetric perturbation of the geometry has to be large enough, and enough cycles of steepest descents have to be applied before the Newton–Raphson iterations. Information about such perturbations may be obtained from an inspection of the eigenvectors corresponding to the M negative eigenvalues of F.)

This procedure for calculating general partial maxima can often reduce or even save the labour and computer time involved in point-by-point mapping calculations, especially if the conformational change includes only one transition state. The partial maximum of the profile of Fig. 2 was approached from point 3 after calculating the points 1, 2 and 3 by the mapping technique (see legend of Fig. 2). Points 4, 5 and 6 were calculated afterwards. The changes of torsion angles on approaching a partial maximum from a suitable trial structure by Newton–Raphson iterations can be very substancial, exactly analogous to the situation when approaching minima.[14] On going to the partial maximum from point 3 of the profile discussed here the torsion angle C(3)–C(4)–C(5)–C(6) changes by 12·4° (excellent convergence; energy increase 0·33 kcal mole^{-1}; see Appendix C).

The one-step calculation of partial maxima as outlined here provides a convenient procedure for the incorporation of experimental conformational barriers into least-squares optimisation calculations of force field parameters.[15]

The calculated activation enthalpy $\Delta H^{\dagger}$ for the crown-distorted chair interconversion of trans-cyclooctene is 10·35 kcal mole^{-1}, for the reverse process 7·21 kcal mole^{-1}. (Some potential energy differences are given in Fig. 2; internal parameters of T, which represents the rate determining barrier, are given in Fig. 1; the imaginary frequency associated with the onedimensional partial maximum of T amounts to $150{\cdot}0\sqrt{(-1)}$ cm^{-1}; the respective eigenvector is the complete reaction coordinate at the transition state and shows the expected large component for the change of the torsion angle C(3)–C(4)–C(5)–C(6). Speculations about a barrier high enough to prevent equilibration of the crown and distorted chair conformations at room temperature[11,16] are therefore strongly discouraged.

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REFERENCES

[1]K. B. Wiberg and R. H. Boyd, *J. Am. Chem. Soc.* **94**, 8426 (1972).
[2]J. W. McIver and A. Komornicki, *Ibid.* **94**, 2625 (1972).
[3]O. Ermer and S. Lifson, *Ibid.* **95**, 4121 (1973).
[4]L. Leiserowitz, private communication; B. S. Green, M. Lahav and G. M. J. Schmidt, *J. Chem. Soc.* (B), 1552 (1971); R. K. Mackenzie, D. D. MacNicol, H. H. Mills, R. A. Raphael, F. B. Wilson and J. A. Zabkiewicz, *Ibid.* (Perkin Trans. II), 1632 (1972); F. A. L. Anet and L. Kozerski, *J. Am. Chem. Soc.* **95**, 3407 (1973).
[5]N. L. Allinger and J. T. Sprague, *Ibid.* **94**, 5734 (1972).
[6]R. L. Hilderbrandt, J. D. Wieser and L. K. Montgomery, *Ibid.* **95**, 8598 (1973).
[7]E. J. Jacob, H. B. Thompson and L. S. Bartell, *J. Chem. Phys.* **47**, 3736 (1967).
[8]J. B. Hendrickson, *J. Am. Chem. Soc.* **89**, 7047 (1967).
[9]N. L. Allinger, M. T. Tribble, M. A. Miller and D. H. Wertz, *Ibid.* **93**, 1637 (1971).
[10]D. N. J. White and G. A. Sim, *Tetrahedron* **29**, 3933 (1973).
[11]O. Ermer, *Angew. Chem, Int. Ed. Engl.* **13**, 604 (1974).
[12]H. M. Pickett and H. L. Strauss, *J. Am. Chem. Soc.* **92**, 7281 (1970).
[13]F. A. L. Anet and A. J. R. Bourn, *Ibid.* **89**, 760 (1967).
[14]D. N. J. White and O. Ermer, *Chem. Phys. Letters* **31**, 111 (1975).
[15]A. Warshel and S. Lifson, *J. Chem. Phys.* **53**, 582 (1970).
[16]R. M. Gavin and Z. F. Wang, *J. Am. Chem. Soc.* **95**, 1425 (1973).
[17]C. Altona and D. H. Faber, *Topics in Current Chem.* **45**, 1 (1974).
[18]M. W. Thomas and D. Emerson, *J. Mol. Structure* **16**, 473 (1973).

APPENDIX A

As a cautionary tale we want to discuss two examples. The first is the C_{2v}-symmetric boat conformation of *cis,cis*-cyclooctadiene mentioned above which has been described as a minimum by Allinger and Sprague.[5]*

The second example is a study of Hilderbrandt *et al.* (HWM) who applied force field calculations to the interpretation of electron diffraction measurements of cyclodecane.[6] These authors calculated the strain energies of 12 symmetric cyclodecane conformations using a steepest descent procedure and a force field based on that of Jacob, Thompson and Bartell.[7] (The conformations had been reported earlier by Hendrickson[8] whose notation is taken over here.) Three conformations contain zero torsion angles due to intersecting perpendicular mirror planes (Long C, CBC, BC). Energy minimisations with our force field[3] led in all three cases to partial maxima the dimensions and imaginary frequencies (magnitudes) of which are as follows: Long C (C_{2h}), M = 2, 222·7, 229·2 cm^{-1}; CBC (C_{2v}), 3, 89·7, 127·9, 235·9; BC (C_s), 2, 95·6, 135·7. In addition, analogous calculations gave partial maxima also for BCC (C_s, M = 1, 85·5 cm^{-1}), CCC (C_{2h}, 2, 42·2, 143·5), and for the symmetric crown (D_{5d}, 4, 126·1, 135·2, each doubly degenerate). Our geometries and relative potential energies (reference: BCB) for the conformations containing partial maxima, except BC, are similar to those of HWM. Excluding TCBC and CBB, the same holds for the 6 remaining true minima. The most serious discrepancies occur for CBB; started with the geometry of HWM the steepest descent process—150 iterations were applied—converged very slowly; only 8 subsequent Newton–Raphson iterations led to a minimum with a concomitant change of 6 torsion angles by about 50–60°, and an energy decrease of about 12 kcal mole^{-1}. Our CBB conformation has a potential energy relative to BCB of 3·95 kcal mole^{-1}, that of HWM 19·74. The average absolute differences between the torsion angles calculated by us and by HWM are 7 and 12° for BC and TCBC, respectively; the corresponding relative energy differences are 20·27 and 3·95 kcal mole^{-1}, respectively, our values being lower. For the other conformations these average differences are 1·8° and 1·76 kcal mole^{-1}.

We therefore believe that a repetition of our calculations using the force field of HWM would hardly affect our conclusions as to the nature of all the cyclodecane conformations dealt with here. A similar remark applies to the C_{2v}-symmetric boat conformation of *cis,cis*-cyclooctadiene-1,5.*

APPENDIX B

Wiberg and Boyd[1] have calculated minimum energy paths for the chair-boat interconversion of cyclohexane using in one calculation a single CCCC-torsion angle as a model for the reaction coordinate. All the points calculated for this coordinate

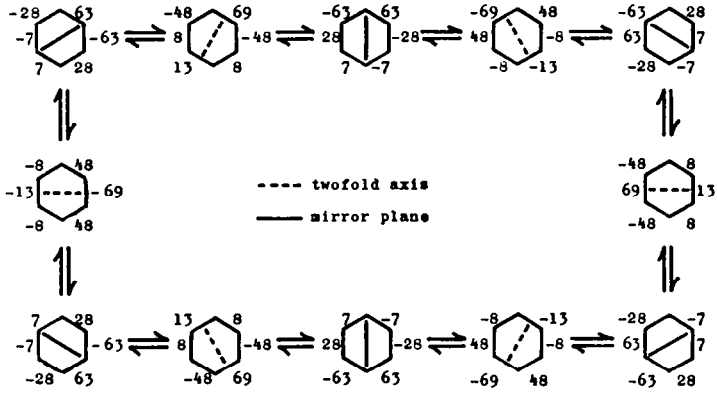
*See note added in proof in section (b).

therefore correspond to C_2-symmetric conformations. We want to remark that a recalculation of this interconversion path with our force field[3] and the technique outlined here results in a C_2-symmetric transition state which in addition to a large imaginary frequency ($248{\cdot}1\sqrt{(-1)}\,cm^{-1}$) shows another very small one ($5{\cdot}8\sqrt{(-1)}\,cm^{-1}$). A second calculation using for the initial point-by-point mapping a HCCH-torsion angle not symmetrically disposed with respect to a twofold axis as the reaction coordinate model, produced a C_s-symmetric transition state with a single imaginary frequency almost identical to the large one of the C_2-transition state ($247{\cdot}4\sqrt{(-1)}\,cm^{-1}$), and a very small yet real frequency ($3{\cdot}9\,cm^{-1}$). Accordingly, the C_s-structure has a potential energy slightly lower than that of the C_2-structure ($\Delta V = 0{\cdot}00086$ kcal $mole^{-1}$). The calculated frequencies of the two structures are very similar (r.m.s. difference $4{\cdot}6\,cm^{-1}$). The small deviations of ΔV and of the two very low frequencies from zero have no physical meaning, but they are certainly significant mathematically as a consequence of the excellent convergence properties of the Newton–Raphson process. This discussion may therefore serve to illustrate that mapping coordinates have to be chosen cautiously.

Our calculations nicely confirm the results of Pickett and Strauss who by analytical methods derived a pseudorotating transition state (quite insensitive to the force field applied) for the chair-boat interconversion of cyclohexane[12] which includes the above mentioned C_s- and C_2-conformations. On the other hand, Wiberg and Boyd[1] obtained a potential energy difference of the two symmetric transition state geometries of 0·5 kcal $mole^{-1}$ (0·6 or 0·8 with another modified force field), in favour of a C_2-structure. (These authors use the terms 'one' and 'two torsional angle activated complex'—abbreviated T_1 and T_2, respectively—for C_2- and C_s-symmetric conformations similar to ours.) We believe that these fairly large energy differences do not reflect properties of the force fields used by Wiberg and Boyd but rather have their origin in the incompleteness of the respective point-by-point mapping calculations. These authors assume for T_1 a geometry with four exactly coplanar consecutive carbon atoms, and for T_2 a geometry with five exactly coplanar carbon atoms. This assumption is, however, only an approximation which, for instance, cannot be justified by the symmetry properties of the two geometries. Our calculated C_2- and C_s-transition state geometries (Table 1) involve CCCC-torsion angles "beyond" the respective zero torsion angles of T_1 and T_2, in accord with earlier calculations of Hendrickson[8]. We feel that a continuation of the mapping calculations of Wiberg and Boyd to torsion angles similar to ours would have led to a further increase in potential energies—to a larger increase for the C_2-form than for the C_s-form—and would have brought the potential energies of both transition state models much closer together. (Our calculated potential energies of T_1 and T_2 are lower than the respective true transition states by 1·00 and 0·45 kcal $mole^{-1}$, and our energy difference between T_1 and T_2 is thus similar to the values of Wiberg and Boyd.) Wiberg and Boyd obtained one imaginary frequency for T_1; for the lowest real frequency they derived a value quite different from zero ($100\,cm^{-1}$), a result not surprising from the foregoing.

We calculate the enthalpy of activation for the inversion process of cyclohexane as 11·07 kcal $mole^{-1}$. ($\Delta V^{\neq} = 12{\cdot}30$; the two very small frequencies of $5{\cdot}8\sqrt{(-1)}\,cm^{-1}$ and $3{\cdot}9\,cm^{-1}$ which characterize the pseudorotational degree of freedom of the transition state are taken as zero and contribute $\frac{1}{2}RT$ to ΔH^{+}. Only the 3N-8 real nonzero frequencies are considered for the evaluation of H_{vibr} of the pseudorotating transition state. The H_{vibr}-values of the C_2- and C_s-structures differ by only 0·02 kcal $mole^{-1}$.) Anet and Bourn[13] measured a value of 10·8 kcal $mole^{-1}$.

The pseudorotational motion of the transition state can be described by the following sequence of 12 sets of torsion angles (deg) for interconverting C_2- and C_s-structures. In each step of this scheme the sign of one torsion angle changes, and the corresponding transient eclipsed partial conformation travels around the 6-membered ring in the fashion "two bonds forward—one bond backward".



APPENDIX C

We used a quadratic constraint function of the type $K_c(\Phi - \Phi_c)^2$ rather than a trigonometric expression[1] for the constrained minimisations. The constraint function contributes only to the

Table 1. Calculated lengths (l), angles (θ), and torsion angles (Φ) for the carbon skeletons of the C_2- and C_s-symmetric transition state conformations of the chair-boat interconversion of cyclohexane (Å, degrees). The torsion angles corresponding to the zero torsion angles of T_1 and T_2 are given in bold type.

C-atom No.		1	2	3	4	5	6
C_2	l		1·529	1·527	1·525	1·529	1·525	1·527
	θ	118·6	118·6	114·8	109·1	109·1	114·8	
	Φ		**−13·1**	−7·7	48·0	−68·7	48·0	**−7·7**
C_s	l		1·528	1·525	1·528	1·528	1·525	1·528
	θ	119·3	116·9	112·2	107·8	112·2	116·9	
	Φ		−7·3	**28·0**	−62·7	**62·7**	−28·0	7·3

derivatives of V, not to V itself. K_c was given a value of 100 kcal mole^{-1} rad^{-2}; the Φ_c-values corresponding to the Φ-values of the mapping points of Fig. 2 deviated from the latter by at most 2·5° (average 1·2°). The minimisations were performed by steepest descent cycles (Eqn 1) followed by Newton–Raphson iterations (Eqn 2). The F-matrix was inverted using a technique in which six suitable Cartesian coordinates are fixed and the corresponding rows and columns of F removed.[17] We want to point out that this very fast procedure is not independent of the choice of the Cartesian coordinate system. For instance, if one chooses the atomic coordinates of three consecutive atoms (1, 2, 3) in a chain as 0, 0, 0; a, 0, 0; b, 0, c and in the programme x_1, y_1, z_1, x_2, z_2 and x_3 are kept constant then the bond length between the first and second atom will not be adjusted, and the bond angle at the second atom will not in general be adjusted properly. (In a case approaching this extreme situation the convergence of the Newton–Raphson process is hampered.) For this choice of six fixed Cartesians it is advisable to transform the coordinates such that those of the three atoms involving fixed Cartesians are as follows: 0, 0, 0; 0, d, 0; 0, e, f. With this provision (which does not solve all the related and rather complex problems; thus difficulties still can occur if a torsion angle involving certain fixed Cartesian coordinates is far from its optimum value) we obtained good results with this method. We did not encounter numerical instabilities as mentioned by Wiberg and Boyd,[1] or problems of the kind described by Thomas and Emerson.[18] Possibly, one of the reasons for this is the fact that, in contrast to these authors, we use analytical first and second derivatives of V. The convergence of the Newton–Raphson technique applied here appears to be practically as good as that of the procedure which makes use of the generalised inverse of F.[15] The time required for the inversion of F by the latter technique (which involves the diagonalisation of F) proved to be 8 times as much as for the direct inversion of the reduced F-matrix. (Telefunken TR 440 computer; trans-cyclo-octene as test molecule; the total time for a Newton–Raphson cycle dropped by a factor of 2·6, i.e. from 42 to 16 sec. This factor increases with molecular size; for a 30-atom system it is 3·2 (91 and 28 sec, respectively).)

The Newton–Raphson iterations leading to the conformations of all constrained and unconstrained minimisations reported here gave for all partial derivatives $\partial V/\partial x_i$ values less in magnitude than 10^{-7} kcal mole^{-1} Å^{-1}.